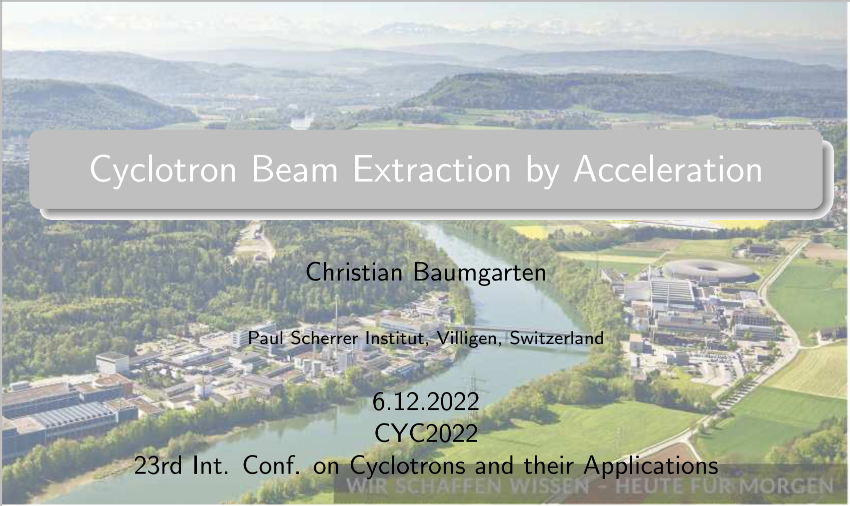

Cyclotron Beam Extraction by Acceleration

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6.12.2022
CYC2022
23rd Int. Conf. on Cyclotrons and their Applications

Outline

Why is Beam Extraction a Problem after all?

- Cyclotron bending limit: $p = q\,R\,B(R)$ is finite.
- It follows that $\frac{dp}{dR} = q\,B\,(1 + \frac{R}{B}\,\frac{dB}{dR}) = q\,B\,(1+k)$.
- Then: Why does the beam (usually) not leave the field when $p > p_{max}$ (i.e. $k \leq -1$)?
- The beam has to pass the fringe field.
- The fringe field is not isochronous.
- The phase shift is (usually) 90° before $p > p_{max}$.
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- We need to reduce the number of turns in the fringe field.
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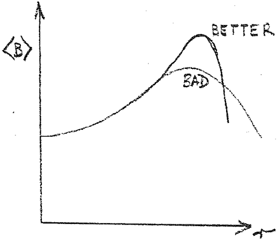

Figure: Taken from: H. Blosser "Modern Cyclotrons", USPAS Lecture 1971

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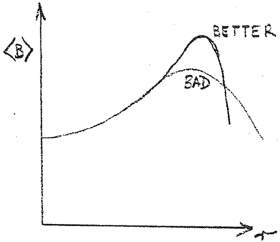

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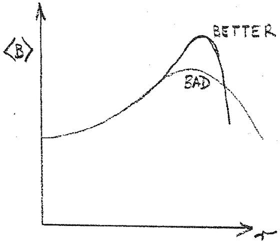

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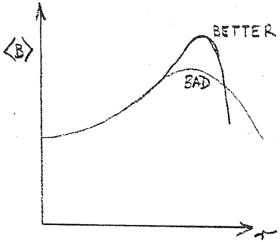

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- A strictly isochronous field is $B_{iso} = B_0\,\gamma$ where $v = \omega_o\,R$ is the velocity and ω_o the circulation frequency.
- $\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-R^2/a^2}}$.
- $a = c/\omega_o$ is called "cyclotron radius".
- Model fringe field by simple Enge function[1, 2] $f(R) = (1 + \exp((R - R_h)/g))^{-1}$
- g is (approx.) pole half-gap, R_h is radius for $f = 1/2$.
- Model: $B(R) = B_0\,\gamma_R\,f(R)$ with $\gamma_R = \frac{1}{\sqrt{1-R^2/a^2}}$.
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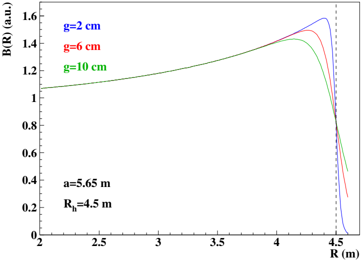
Example similar to PSI Ring

- The cyclotron radius a is determined by circulation frequency.
- The half-$f(R)$ radius R_h is determined by the extraction energy.
- *Rightarrow* Field model has only one free parameter g:

$$B(R) = B_0\,\gamma_R\,f(R)$$

$$\gamma_R = \frac{1}{\sqrt{1 - R^2/a^2}}$$

$$f(R) = \frac{1}{1 + \exp\left((R - R_h)/g\right)}$$



- Parameter g is approximately equal to half of the pole-gap.

Some Calculations

- Compute derivative

$$\frac{df}{dR} = -\frac{1}{g}\,f\,(1-f)$$

to obtain field index:

- $$k = \frac{R}{B}\,\frac{dB}{dR} = \gamma_R^2 - 1 - (1-f)\,\frac{R}{g}$$

- with revolution frequency ω_{rev}:

$$\omega_{rev} = \frac{q}{m\,\gamma}\,B(R) = \frac{q}{m\,\gamma}\,B_0\,\gamma_R\,f(R) = \frac{\gamma_R}{\gamma}\,\omega_0\,f(R)$$

- where “nominal frequency” $\omega_0 = \frac{q}{m}\,B_0$.

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The physical γ-factor

Use real velocity

$$v = \omega_{rev}\, R = R\, \frac{\gamma_R}{\gamma}\, \omega_0\, f(R)$$

to compute physical γ-factor:

$$\gamma = \frac{1}{\sqrt{1 - (\frac{R}{R_\infty}\, \frac{\gamma_R}{\gamma}\, f)^2}}$$

Solve for γ:

$$\gamma = \sqrt{1 + (\gamma_R^2 - 1)\, f^2}$$

... or for γ_R, respectively:

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Compute Phase Shift (1)

ϕ is the phase when the bunch crosses the gap.

- Phase shift per radius increase: $\frac{d\phi}{dR} = \frac{d\phi}{dn}\,\frac{dp}{dR}\,\frac{dE}{dp} / \frac{dE}{dn}$
- Phase shift per turn: $\frac{d\phi}{dn} = 2\,\pi\,N_h\,(\frac{B_{iso}}{B} - 1) = 2\,\pi\,N_h\,(\frac{\gamma}{\gamma_R\,f} - 1)$. where N_h is "harmonic number".
- with energy gain per turn $\frac{dE}{dn} = q\,V_{rf}\,\cos(\phi)$
- $\frac{dE}{dp} = v$ (Hamilton's equation of motion).
- From above: $\frac{dp}{dR} = q\,B\,(1 + k)$
- Put together: $\frac{d\phi}{dR} = \frac{2\,\pi\,N_h\,m\,c^2}{q\,V_{rf}\,\cos(\phi)}\,\frac{R}{a^2}\,(1 - \frac{\gamma_R\,f}{\gamma})\,\gamma_R\,f\,(1 + k)$
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- $\Rightarrow$ Change integration variable from R to f.
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- Then $R \approx R_x \approx \mathrm{const}$ and $\gamma_R \approx \gamma_x \approx \mathrm{const}$.

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$$\sin(\phi_f) = \frac{2\,\pi\,N_h\,m\,c^2\,\gamma^3}{2\,q\,V_{rf}}\,\frac{g^2}{a^2} \leq 1$$

- The minimal voltage is then (using $\beta \approx R/a$):

$$q\,V_{rf} \geq \pi\,N_h\,m\,c^2\,\gamma\,(\gamma+1)\,(\gamma-1)\,\frac{g^2}{R^2}$$

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PSI Medical Cyclotron "COMET"

$$\frac{q\,V_{rf}}{E} \geq \pi\, N_h \gamma\,(\gamma+1)\,\frac{g^2}{R^2}$$

- $E = 250\,\mathrm{MeV} \Rightarrow \gamma = 1.266$.
- Harmonic number $N_h = 2$.
- Half-Gap: $g \approx 2.2\,\mathrm{cm}$.
- Extraction radius $R = 0.81\,\mathrm{m}$.
- Minimal voltage for beam escape: $q\,V_{rf} \geq 3.3\,\mathrm{MeV}$.
- Energy gain per turn is $0.384\,\mathrm{MeV}$.
- For COMET, the voltage is either an order of magnitude too low or the gap is a factor of ≈ 3 too large for beam escape.
- With a half-gap of $g \leq 7\,\mathrm{mm}$, beam "escape" might become possible.

(IBA self-extracting cyclotron has a half-gap of $7.5\,\mathrm{mm}$ [3].)

PSI Ring: Escape Extraction

Main result:

$$\frac{q\,V_{rf}}{E} \ge \pi\,N_h\gamma\,(\gamma+1)\,\frac{g^2}{R^2}$$

- $E = 590\,\mathrm{MeV} \Rightarrow \gamma = 1.63$.
- Harmonic number $N_h = 6$.
- Half-Gap: $g \approx 2\,\mathrm{cm}$.
- Extraction radius $R = 4.5\,\mathrm{m}$.
- Minimal voltage for beam escape: $q\,V_{rf} \ge 940\,\mathrm{keV}$.
- Today's energy gain per turn is $2.6\,\mathrm{MeV}$.
- Beam escape condition is likely fullfilled in PSI Ring machine!
- Next: Verify by numerical tracking.

Opening the $E - \phi$-Loop: (Starting with $\phi = 0$ before extraction $E = 530\,\mathrm{MeV}$)

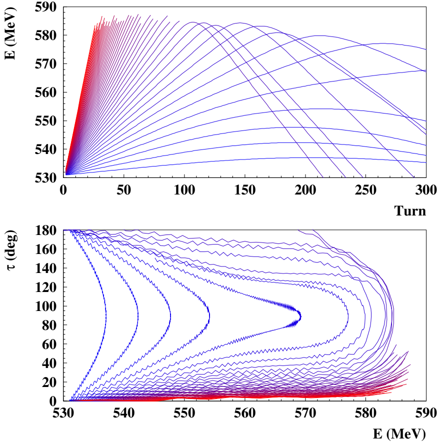

Figure: Top: Energy vs. Turn Number. Bottom: RF-phase vs. energy. From blue to red: Increasing energy gain.

$(590\,\mathrm{MeV} - 530\,\mathrm{MeV})/90 \geq 670\,\mathrm{keV/turn}$ for escape extraction.

Accuracy: How to determine g?

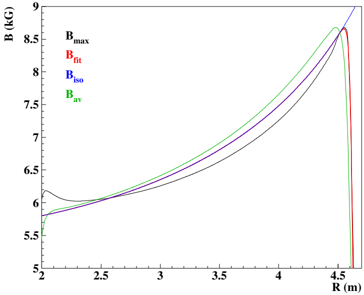

Figure: Fit parameter g to $B_{max}(R)$ (max. field at radius R) gives $g = 15\,\mathrm{mm}$ and therefore $q\,V_{rf} \geq 530\,\mathrm{keV}$. Fit parameter g to $B_{av}(R)$ (average field at radius R) gives $g = 36\,\mathrm{mm}$ and therefore $q\,V_{rf} \geq 3\,\mathrm{MeV}$.

- Several approximations were used, mo important is $g \ll R$.
- Strictly speaking, g describes the field shape and is only "of the order" of half of the pole gap.
- Due to orbit scalloping, a fit to the average field overestimates g.
- A fit to the maximum field, on the other hand, can underestimate g.
- Is g really half of the air-gap D?
- Enge gives two examples [1, 2], where $g \approx D/2.388$ and $g \approx D/2.341$, depending on exact pole shape. We simply assumed $g \approx D/2$.
- Accuracy of formula probably not better than 50 %, mostly due to uncertainties in value of effective half-gap size g.

PSI Ring: Need for Gradient Correctors (1)

Need for gradient correctors, calculated for PSI Ring cyclotron:

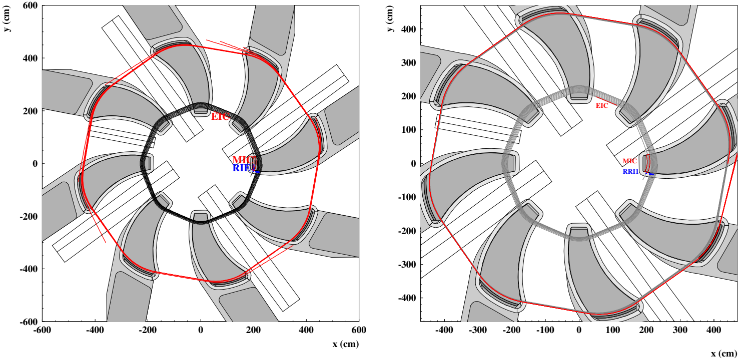

Figure: Some tracks with slightly different starting conditions in R, P_r, E and rf phase τ. Left: Without gradient corrector. Right: With passive magnetic channels as gradient correctors.

PSI Ring: Gradient Correctors (2)

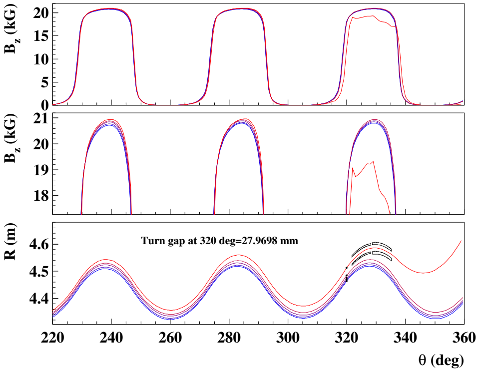

Figure: The last turns of the ring cyclotron with gradient corrector. Turn number (and energy) increases from blue to red. Top/middle: Field seen by the orbit. Bottom: Radius vs. Angle.

- Fast extraction: The field gradient is positve or zero up to the last sector.
- Gradient corrector is required, but only on one sector.
- Actual turn separation 18 mm [6], by acceleration with gradient corrector 28 mm.
- Note: Gradient corrector generates first harmonic (not compensated).

Electrostatic Extractor vs. Acceleration (1)

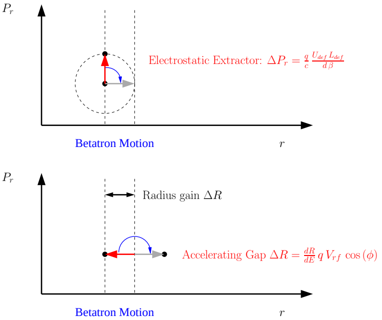

Figure: Schematic vizualization of Extraction methods. $r - P_r$-circles with actions of electrostatic extractor versus acceleration gap.

- Electrostatic extractor induces change in radial momentum ΔP_r.
- Fast strong acceleration induces $\Delta R = \frac{dR}{dE}\,\Delta E$.
- Betatron motion converges both kicks into a radius increase after $\Delta\psi = \pi/4$ and $\Delta\psi = \pi/2$, respectively.

Electrostatic Extractor vs. Acceleration (2)

- Electrostatic extractor (EE): $\Delta P_r = \frac{q\,U_{def}\,L_{def}}{v_\theta\,d_{def}}$ so that: $x' \approx \frac{\Delta P_r}{P_\theta} = \frac{L_{def}}{d_{def}}\,\frac{q\,U_{def}}{m\,c^2\,\gamma\,\beta^2}$.
  - PSI Ring: $L_{def} \approx 60\,\mathrm{cm}$, $d_{def} \approx 2\,\mathrm{cm}$, $U_{def} = 150\,\mathrm{kV}$.
  - $\Rightarrow$ $\Delta x' \approx 4.5\,\mathrm{MV}\,\frac{q}{m\,c^2\,\gamma\,\beta^2}$.
- Acceleration $\Delta R = \frac{dR}{dE}\,\Delta E \approx \frac{q\,V_{rf}\,\cos\phi\,R}{m\,c^2\,\gamma\,\beta^2\,(1+k)}$.
  - Acceleration: $\phi \leq 10^\circ$, $V_{rf} \approx 3\,\mathrm{MV}$, $k \approx 0$.
  - $x' = \frac{2\Delta R}{\pi\,R/\nu_r}$
  - $\Rightarrow$ $x' \approx 2\,\mathrm{MV}\,\frac{q}{m\,c^2\,\gamma\,\beta^2}$
- Kick by acceleration reaches same order of magnitude as EE.
- But in the PSI Ring, the Kick by EE still stronger than "kick by acceleration".
- However, the higher the energy gain/turn, the less EE contribute to extraction.

Dream Machines (1)

"Dream machine": Multi-MW high power proton cyclotron for ADS, spallation sources and/or neutrino physics.

- Recall "Joho's N^3-Law" [4]: the possible beam current (at constant losses) scales with the $1/N^3$ where N is the number of turns.
- This translates to: $I_{max} \propto V_{rf}^3$.
- Hence high intensity cyclotrons require an energy-gain/turn as high as possible.
- Two options: 1) Increase of cavity voltage or 2) increase number of cavities.

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The energy efficiency is [5, 6]:

$$\eta_{acc} = \frac{P_{beam}}{N_{cav}\, V_{cav}^2/(2R_{cav}) + P_{beam}/\eta_{rf} + P_{aux}} \,.$$

- $\Rightarrow$ an increase of the number of cavities increases ohmic losses linearily. An increase of the cavity voltage increases ohmic losses quadratically.
- Hence a large number of cavities is prefered over a super-high cavity voltage. (Costs more money though...)
- Large number of cavities requires a large number of sectors.
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- Large number of cavities requires a large number of sectors.
- A large number of sectors requires a large radius (but not a large gap).
- $\Rightarrow$ the possibility for "escape-extraction" is included in such "dream machines" for "free".
- True "Dream Machines" need no (and should have no) electrostatic extractor.

Dream Machines (2)

The energy efficiency is [5, 6]:

$$\eta_{acc} = \frac{P_{beam}}{N_{cav}\, V_{cav}^2/(2R_{cav}) + P_{beam}/\eta_{rf} + P_{aux}}\,.$$

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Summary

- We estimated the total shift of the beam phase in the course of the extraction process using a simple Enge-type fringe field model.
- "Self-Extraction" has been proposed before[3], but (to our best knowledge), the condition for "self-extraction" has never been derived before.
- However: "Self-Extraction" does not imply that there are no extraction elements needed in practice.
- The strong (negative) field gradient the beam passes during extraction requires compensation by "gradient correctors" and/or special field shaping[3].
- Furthermore the design requirements of high power separate sector cyclotrons facilitate beam escape by acceleration.
- Then it should be possible to avoid electrostatic extractors in "dream machines"..

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Thank You.

Publication in progress [7].

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PSI Ring: Betatron enhanced Turn Separation (1)

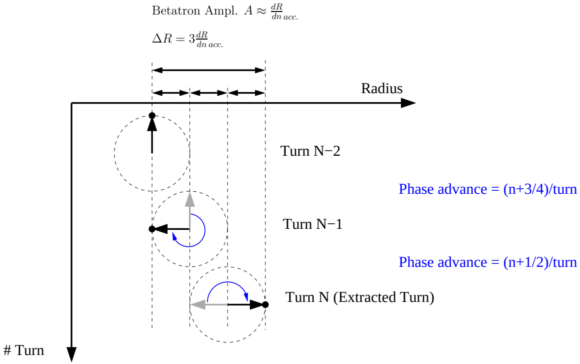

Figure: Simplified picture of betatron enhanced turn separation, assuming a constant radius gain.

However, if phase shift in the fringe field is negligible ($|\phi| < 15^\circ$ or so), then the radius gain depends mostly on radial tune ν_r:

$$\frac{dR}{dE} = \frac{R\,\gamma}{E\,(\gamma+1)}\,\frac{1}{\nu_r^2}$$

PSI Ring: Betatron enhanced Turn Separation (1)

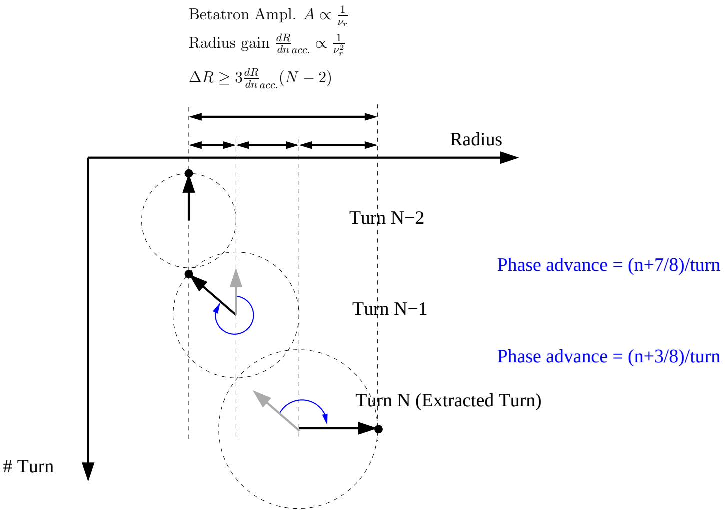

Figure: Realistic picture of betatron enhanced turn separation with increasing radius gain.

PSI Ring: Betatron enhanced Turn Separation (3)

If the phase shift during extraction is as low as in PSI Ring cyclotron, then the energy gain is approximately constant during extraction.

- Then the radius gain is proportional to $1/\nu_r^2$
- $\frac{dR}{dn}$ increases to infinity *because* $\nu_r \to 0$.
- Betatron enhancement does *not* require a specific ν_r.
- Several possible schemes exist for betatron enhanced turn separation at extraction, specifically if $\nu_r > 1.5$ prior to extraction.